# Newton's Laws of Motion

Now that we have learned how to **describe** motion, how do we **cause** the motion that we want?

We apply **forces** on an object!

But what do forces directly affect:

location?   velocity?   acceleration?   jerk?

Newton answered these questions by postulating **three** laws of motion.

# Newton's First Law of Motion

**Newton's First Law of Motion**: an object in motion will, in the absence of forces, tend to remain in motion with neither the speed nor direction changing.

This, at first, does not seem obvious. Most things on earth tend to slow down and stop. However, when we consider the situation, we see that there are lots of forces tending to slow the objects down such as friction and air resistance.

# Newton’s First Law of Motion

When we look at the planets and moon, however, it is easier to see that they just keep right on going!

Also, when we remove or reduce a lot of the forces on an object, it does tend to keep right on going.  Consider a ball rolling on a smooth floor.  We don’t need forces to keep the motion going!

# Newton's Second Law of Motion

If we want to **change** the motion, we push on it (that is, apply forces).

Newton states this in his **Second Law of Motion**: The resultant force (vector sum of the individual forces) on an object causes the object to accelerate in the same direction as the resultant force and in inverse proportion to the mass of the object:

$$\Sigma \mathbf{F} = m\mathbf{a} \, .$$

# Newton's Second Law of Motion

Note that this is a **vector** equation, and should really be worked in component form:

$$\boldsymbol{\Sigma\, F_x = ma_x}$$

$$\boldsymbol{\Sigma\, F_y = ma_y}\ .$$

We can now see that Newton's First Law of Motion is really just a special case of his Second Law of Motion.

# Newton's Third Law of Motion

There is one further important aspect of motion that Newton identified: the distinction between forces that act **on** an object and forces that act **by** the object. This leads to his **Third Law of Motion**: **For every force by a first object on a second object, there is a force by the second object on the first object with the same magnitude but in the opposite direction.**

# Newton’s Third Law of Motion

This is sometimes called the law of action and reaction.

I like to call it: **you can’t push yourself**! You can only push on an object and hope that it pushes back.

Example: when you walk up a stairs, you use your muscles to push down on the stairs and you trust that the stairs will push back up on you lifting you up the stairs.

# Mass

Here for the first time we encounter **mass**. Note that mass relates acceleration to resultant force: the bigger the acceleration for the same force, the smaller the mass. This property of matter is actually called **inertial mass**.

We did not need mass when considering the description of motion, but we do need mass when considering how to cause that motion using forces.

# Units

The units of mass are **kilograms**.  This is the third fundamental unit (along with meters and seconds) in the MKS system of units.

The units of force in the MKS system are **Newtons**, where a force of 1 Nt will give a mass of 1 kg an acceleration of 1 $m/s^2$ .

# Forces

In order to work with forces, we have to identify the common forces we find, both as to magnitude and direction:

- **gravity** (near earth's surface, this is called weight, W) **magnitude** = **m*g**; **direction** = **down**

Note that **mass** is involved in the force of gravity! This is a separate property from that of inertia, so we give this property the name **gravitational mass**.

# Forces

Since **mass** is involved on both sides of Newton's second law when gravity is the only force (falling object): in $\mathbf{F_{gr} = -mg}$ as gravitational mass and in $\mathbf{m*a}$ as inertial mass, the mass cancels out giving us the reason all objects fall with the same acceleration (neglecting air resistance)!

$$\mathbf{\Sigma F = ma} \quad \text{becomes:} \quad \mathbf{-mg = ma}$$

# Forces

Later we'll look more closely at gravity, even when we are not near the earth's surface.

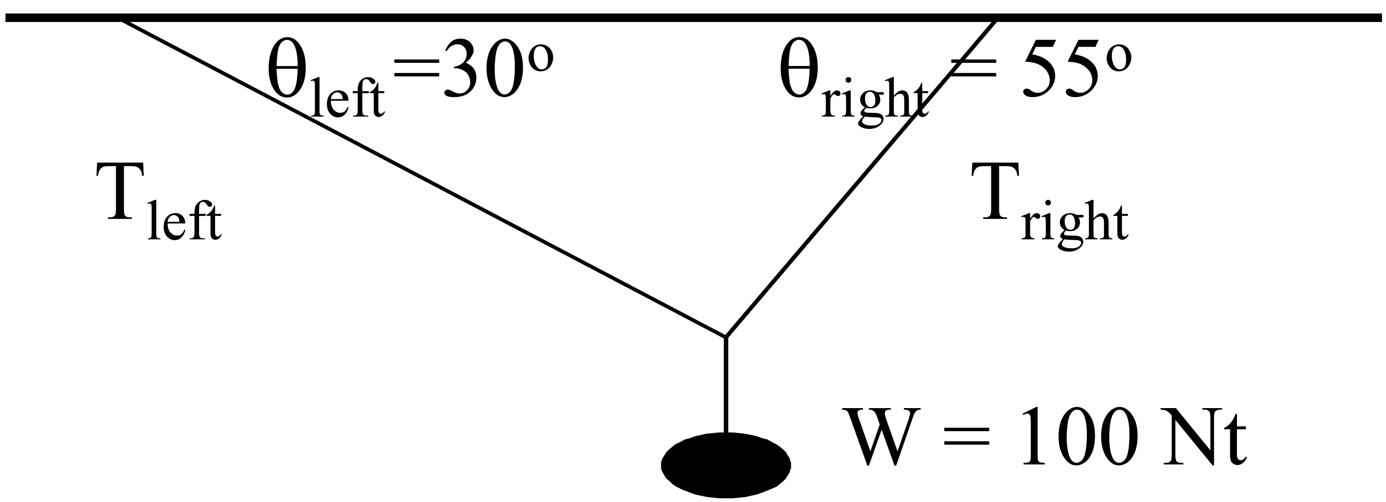
- **contact force**, $F_c$: **magnitude** = **balances up to point of collapse**; **direction** = **perpendicular to the surface** that supplies the contact.

# Forces, cont.

- **friction**, $F_f$: **magnitude**: **balances** up to a point, and then reaches a **constant value** that depends on the two surfaces and how hard the two surfaces are being pressed together ( $F_f \leq \mu F_c$ ), **direction: parallel to surface**.
- **tension**, T: **magnitude: pulls same at one end as another** unless rope is being accelerated; **direction: parallel to rope**.

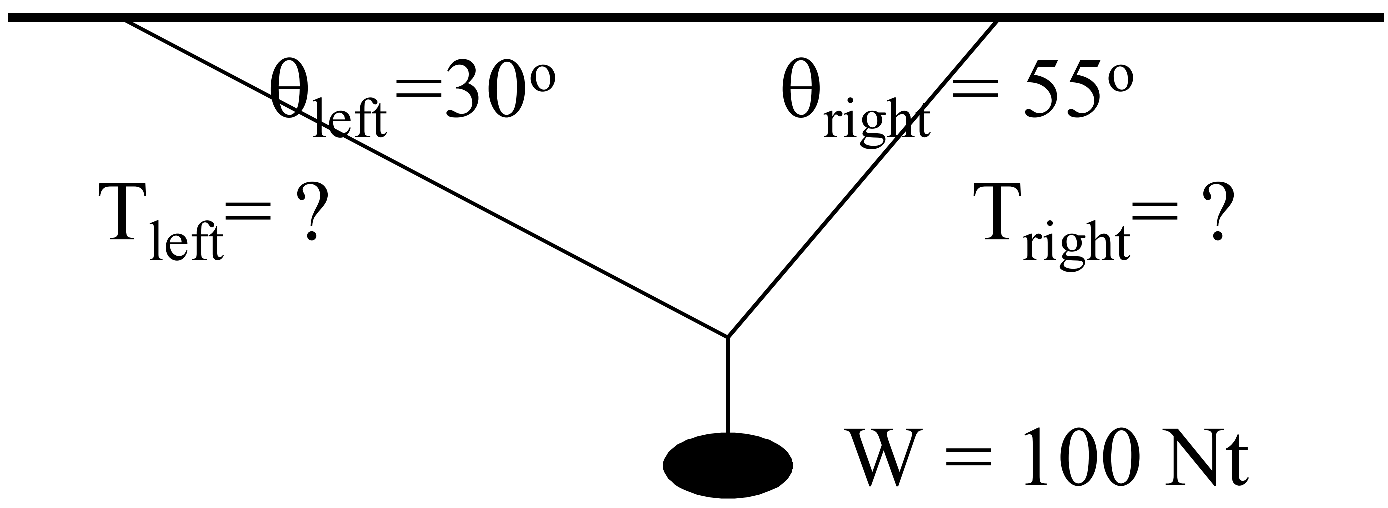
# Statics

- **Statics** is the name for situations in which there is **zero acceleration**.
- **Example**: consider the situation below where two ropes hold up a weight:



# Example of Statics

- What is the tension in each rope?
- Are the two tensions the same, or, if not, which rope has the higher tension?



# Statics Example

- We have the diagram, and we have the information on the diagram.
- We know what we're looking for ($T_{left}$ and $T_{right}$).
- What principle or law do we employ to relate what we know to what we don't know?

# Statics Example

- We recognize this as a statics problem (since there is no motion and hence no acceleration). Thus we have:

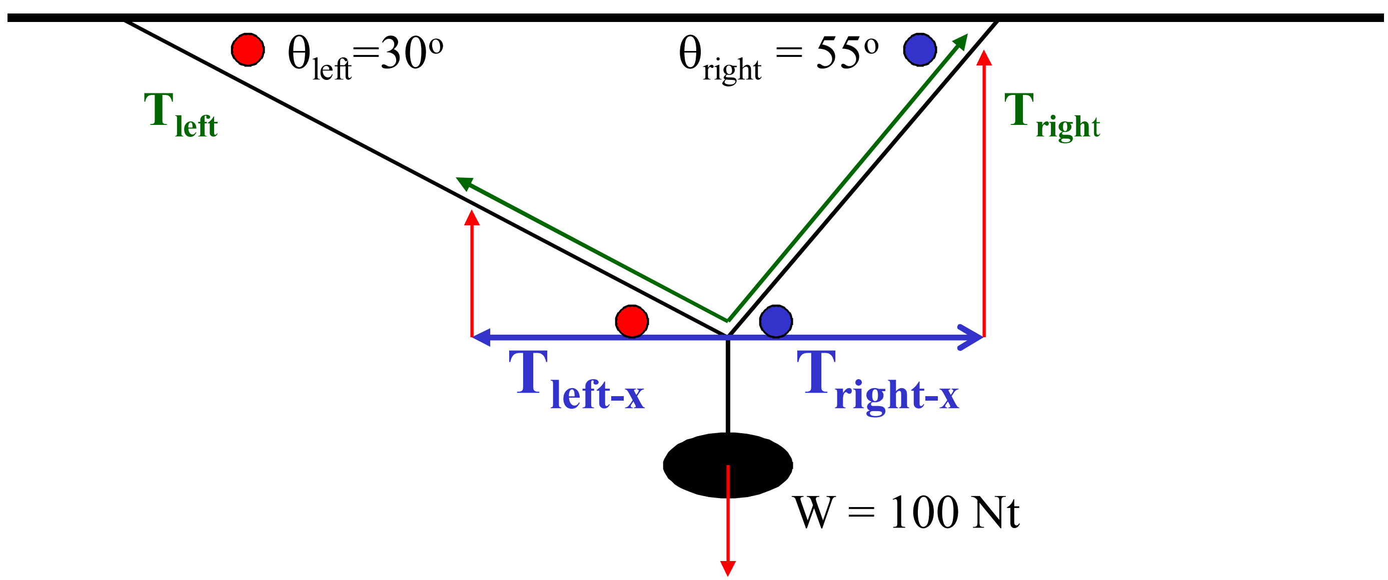
$$\mathbf{\Sigma\ F_x = 0}$$

$$\mathbf{\Sigma\ F_y = 0.}$$

- We are given three forces (W, $T_{left}$ and $T_{right}$) in **polar** form, so we need to convert these polar vectors into **rectangular** form.

# Example of Statics

- $\mathbf{T_{left\text{-}x}} = -\,\mathbf{T_{left}} \cos(\theta_{left}) = -\,\mathbf{T_{left}} \cos(30^{o})$
- $\mathbf{T_{right\text{-}x}} = +\,\mathbf{T_{right}} \cos(\theta_{right}) = +\,\mathbf{T_{right}} \cos(55^{o})$
- $\mathbf{W_x} = 0$



# Example of Statics

- Note that, since $W_x = 0$, $\mathbf{T}_{\mathbf{left\text{-}x}}$ must be the same as $\mathbf{T}_{\mathbf{right\text{-}x}}$ (except for sign).
- Also note that the x components do nothing to help hold up the weight, since the weight acts strictly in the negative y direction!
- From this fact and using the diagram, we can see that the magnitude of $\mathbf{T}_{\mathbf{right}}$ , since it is steeper, must be greater than that of $\mathbf{T}_{\mathbf{left}}$.

# Example of Statics

- From Newton's Second Law for the x components we have then:

$\Sigma F_x = - \mathbf{T}_{\mathbf{left}} \cos(30^o) + \mathbf{T}_{\mathbf{right}} \cos(55^o) + 0 = 0$

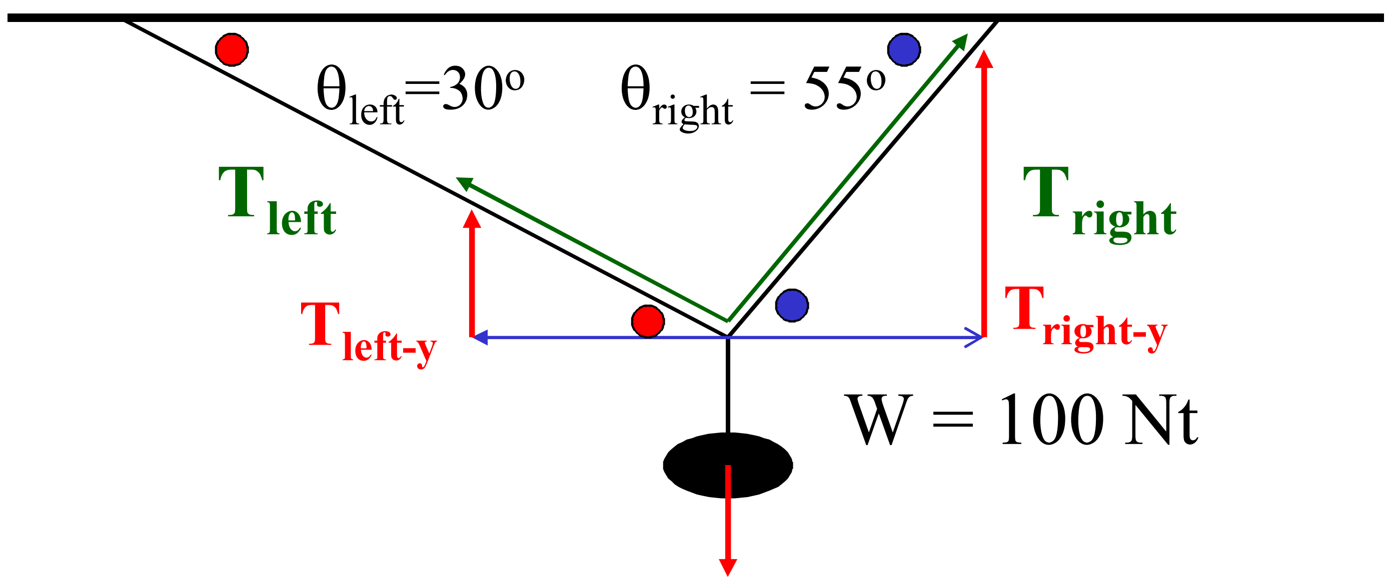
or simplifying:

$$\mathbf{T}_{\mathbf{left}} \cos(30^o) = \mathbf{T}_{\mathbf{right}} \cos(55^o) \,.$$

- This is one equation in two unknowns ($\mathbf{T}_{\mathbf{left}}$ and $\mathbf{T}_{\mathbf{right}}$), so we also need to consider the y component equation.

# Example of Statics

- $T_{left-y} = + T_{left} \sin(\theta_{left}) = + T_{left} \sin(30^o)$
- $T_{right-y} = + T_{right} \sin(\theta_{right}) = + T_{right} \sin(55^o)$
- $W_y = -\ 100$ Nt



# Example of Statics

- From Newton's Second Law for the y components we have then:

$\Sigma F_y = + \mathbf{T}_{\mathbf{left}} \sin(30^o) + \mathbf{T}_{\mathbf{right}} \sin(55^o) - 100$ Nt $= 0$

or simplifying:

$$\mathbf{T}_{\mathbf{left}} \sin(30^o) + \mathbf{T}_{\mathbf{right}} \sin(55^o) = 100 \text{ Nt} .$$

- This is one equation in two unknowns ($\mathbf{T}_{\mathbf{left}}$ and $\mathbf{T}_{\mathbf{right}}$), so we have two simultaneous equations to solve.

# Example of Statics

$$T_{left}\cos(30^o) = T_{right}\cos(55^o)$$

$$T_{left}\sin(30^o) + T_{right}\sin(55^o) = 100\ \text{Nt}$$

- Solving the first equation for $T_{left}$ in terms of $T_{right}$: $T_{left} = T_{right}\cos(55^o) / \cos(30^o)$
- And using this in the second equation gives

$$[T_{right}\cos(55^o) / \cos(30^o)]\sin(30^o) +$$

$$T_{right}\sin(55^o) = 100\ \text{Nt}$$

# Example of Statics

$$[\mathbf{T_{right}} \cos(55^o) / \cos(30^o)] \sin(30^o) +$$

$$\mathbf{T_{right}} \sin(55^o) = 100\ \text{Nt}$$

This is now one equation for one unknown ($T_{right}$), so we have: $T_{right}$ =

100 Nt / [cos(55$^o$)*sin(30$^o$)/cos(30$^o$) + sin(55$^o$)] = **86.93 Nt**.

We now use $T_{left} = T_{right} \cos(55^o) / \cos(30^o)$

to get $T_{left}$ = 86.93 Nt * cos(55$^o$) / cos(30$^o$) = **57.58 Nt**.

# Example of Statics

$\mathbf{T_{right} = 86.93\ Nt}$.  $\mathbf{T_{left} = 57.58\ Nt}$

Note that $T_{right}$ is in fact larger than $T_{left}$ as we figured it should be from the diagram earlier.

Also note that the sum of the magnitudes of $T_{right}$ and $T_{left}$ are **greater than** the weight of 100 Nt. This is because part of each tension, the x-component, goes into pulling sideways instead of pulling up.

# Dynamics

When things do move in response to forces, we have what we call dynamics. In dynamics there is an acceleration on the object.

If all the forces are constant during the motion, then we can use the equations we have for constant acceleration. If the motion is circular, then we can use the equations we have for circular acceleration.

Let’s now consider a common situation: riding an elevator.

# **Example:** Elevator

You are in an elevator at rest on the ground floor. You stand on a scale. What does the scale read? Assume for definiteness sake that your mass is 70 kg (about my mass).

# Elevator

Standing on a scale in an elevator at rest (or in constant motion), we have a statics problem in one dimension: $\Sigma F_y = +F_{scale} - mg = 0$.

The scale reads the contact force that the scale is exerting. In this case, it is easy to see that the force of the scale balances your weight, and so the scale reads your weight:

$$\mathbf{F_{scale} = mg = 70\ kg * 9.8\ m/s^2 = 686\ Nt.}$$

# Elevator

Now the elevator starts moving - dynamics. When this happens, the scale reads **730 Nt** (which is more than your weight of 686 Nt).

Is the elevator moving up, moving down, or can you tell?

What is the acceleration of the elevator?

# Elevator

What we have is still based on **Newton's Second Law**: in this case, however, although your weight hasn't changed, the force the scale exerts has increased. This must mean that there is an acceleration. Since the force of the scale is greater than your weight, the net force is up, and so the acceleration is up. And since you were at rest, you now must be moving up!

# Elevator

$\mathbf{\Sigma F_y = +F_{scale} - mg = ma_y}$

730 Nt- 686 Nt $=$ 70 kg * $a_y$ or

$a_y = 44$ Nt / 70 kg $= +0.63\ \mathrm{m/s^2}$ .

If the acceleration is positive (and the force of the scale is larger than your weight), are you always moving up (positive)?

# Elevator

Not necessarily. A positive acceleration only means the velocity is changing and becoming: either more positive, or less negative.

Besides moving up and speeding up, you could be moving down but slowing down. In fact, when you are going down in an elevator, just before you reach the bottom floor, you do feel "heavy".

# More Examples

There are more examples of using basic forces in Newton's Second Law in the **Computer Homework Assignment, Vol 1 #7**, entitled Newton's Second Law.

# Gravitational Force

- Previously we saw that the force of gravity depended on the mass of an object as well as the constant acceleration due to gravity, g.
- But we know that objects on the moon fall to the moon’s surface, not to the earth’s surface. If objects do have mass, then why don’t ALL objects fall to the earth?

# Gravitational Force

- Newton "discovered" the Law of Gravity by sitting under an apple tree and getting conked! Is this correct?
- Imagine sitting under an apple tree and getting conked by an apple. Also consider that you see the moon up in the sky. Have you wondered why the apple falls down but the moon doesn't?

# Gravitation

- But what makes the moon go around the earth instead of continuing off into space?
- If the moon is orbiting, there must be some force causing the circular acceleration for circular motion.  The obvious answer (at least now) is that the earth's gravity does cause the moon to fall - it's just moving sideways so that it continues to move and fall - continues in its circular orbit!

# Newton’s Law of Gravity

- In looking at gravity as the cause of the moon’s circular motion, Newton came to the conclusion that the force of gravity had to be weaker at the moon’s distance than it was on the earth - otherwise the moon would have to be going much faster to stay in orbit.

# Newton's Law of Gravity

- Newton came up with the following equation for gravity: any two objects attract one another based on the mass of each, the distance apart, and some constant based on units: $\mathbf{F_{gravity} = G*m_1*m_2 / r_{12}^2}$

where $\mathbf{r_{12}}$ is the **distance between $m_1$ and $m_2$** and $\mathbf{G = 6.67 \times 10^{-11}\ Nt*m^2 / kg^2}$ which describes how strong gravity is.

# Newton’s Law of Gravity

Do all objects with mass attract all other objects with mass? Are you attracted to your neighbor (gravitationally, that is)?

We have done experiments that show the answer is **yes**!

# Newton's Law of Gravity

However, because the strength of gravity is very weak, the force of attraction is very weak.  For one kilogram separated from another kilogram by one meter:

**$F_g = Gm_1m_2/r_{12}^2$** =

(6.67 x $10^{-11}$ Nt-$m^2$/$kg^2$)*(1 kg)*(1 kg) / [1 m]$^2$

**= 6.67 x $10^{-11}$ Nt**   (a very small force).

# Newton's Law of Gravity

At the earth's surface, we have

$$F_{gravity} = G*M_{earth}*m / R_{earth}^2$$

where the distance between an object on the earth's surface and the earth (center to center distance) is the radius of the earth.

Note that G, $M_{earth}$ and $R_{earth}$ are all constant, so that near the earth this reduces to

$F_{gravity} = mg$ where $g = G*M_{earth}/R_{earth}^2$ .

# Mass of the Earth

The great gravity we feel on the earth is due to the huge mass of the earth. Even though gravity is weak, the huge mass of the earth combines lots of very weak forces into one reasonably strong force.

But how much mass does the earth have?

# Mass of the Earth

We can use the equation: **$g = G*M_{earth}/R_{earth}^2$**

to solve for $M_{earth}$ since we know

**$g = 9.8\ m/s^2$** (from our lab experiment),

**$G = 6.67 \times 10^{-11}\ Nt\text{-}m^2/kg^2$** (from precise gravity force experiments), and

**$R_{earth} = 6{,}400\ km$** (since we know the circumference of the earth = 25,000 miles).

# Mass of the Earth

$\mathbf{g = G^*M_{earth}/R_{earth}^{\ 2}}$ or $\mathbf{M_{earth}} = g^*R_{earth}{}^2/G$

$= 9.8\ m/s^2 * (6.4 \times 10^6\ m)^2 / 6.67 \times 10^{-11}\ Nt\text{-}m^2/kg^2$

$= \mathbf{6.0 \times 10^{24}\ kg}$ .

This value is certainly large as we expect the mass of the earth to be large. But is there **another way** to get the same answer? If there is, that would greatly add to our confidence in our answer!

# Another way

How about using the fact that the moon orbits the earth - due to the earth's gravity?

We know that the moon goes in (roughly) a circular orbit, and we can use Newton's Second Law to relate that circular orbit to the Earth's gravity: $\mathbf{F_{gravity} = m_{moon}\ a_{circular}}$

$GM_{earth}m_{moon}/R_{earth\text{-}moon}^{2} = m_{moon}\omega^2 R_{earth\text{-}moon}$

or $\mathbf{M_{earth} = \omega^2 R_{earth\text{-}moon}{}^3/G}$ .

# Another Way

$\mathbf{M_{earth} = \omega^2 R_{earth\text{-}moon}{}^3/G}$

where $\omega = 2\pi f = 2\pi/T$ and

$T = 1$ month (actually 27.3 days) $= 2.36 \times 10^6$s

$R_{earth\text{-}moon} = 250{,}000$ miles $= 3.84 \times 10^8$ m

$\mathbf{M_{earth}} = (2\pi/2.36 \times 10^6 \text{s})^2 * (3.84 \times 10^8 \text{ m})^3 /$
$6.67 \times 10^{-11} \text{ Nt-m}^2/\text{kg}^2 \;=\; \mathbf{6.0 \times 10^{24}\ kg}$

which matches the first answer for $m_{earth}$ !

# Satellites

The same concepts (equations):

$$\mathbf{F_{gravity} = m_{sat}\ a_{circular}}$$

where $\mathbf{F_{gravity} = GM_{earth}m_{sat}/R_{sat}^2}$ and

$\mathbf{a_{circular}} = \omega^2\mathbf{R_{sat}}$ , $\mathbf{v_\theta} = \omega^2\mathbf{R_{sat}}$ , $\omega = 2\pi$**f, and f = 1/T**

can be used to determine the period of a satellite in circular orbit around the earth, or determine the radius the satellite needs to have for a certain desired period (such as T=24 hours for a geosynchronous satellite).

# Space Shuttle

- The same relations can be used to get the necessary speed of the space shuttle in its orbit.  All we need are those equations plus something about the orbit (height or period).
- **Problem:**  If the space shuttle orbits at a height of 200 miles (=  320 km) above the earth's surface, how fast does it need to be going in its orbit?

# Space Shuttle

Since we have an orbiting satellite (in this case, the shuttle acts like the satellite):

$$\mathbf{F_{gravity} = m_{sat}\, a_{circular}}$$

where $\mathbf{F_{gravity} = GM_{earth}m_{sat}/r_{sat}^2}$ and

$\mathbf{a_{circular} = \omega^2 R_{sat}}$ , $\mathbf{v_\theta = \omega^2 R_{sat}}$ , $\mathbf{\omega = 2\pi f}$, **and** $\mathbf{f = 1/T}$.

We know G, $M_{earth}$, and $\mathbf{r_{sat} = R_{earth} + height}$, and we note that the $m_{sat}$ will cancel out (that is, $m_{sat}$ is irrelevant!).

# Space Shuttle

$F_{gravity} = GM_{earth}m_{sat}/r_{sat}^2 = m_{sat}\, a_{circular} = m_{sat}\omega^2 r_{sat}$

or $GM_{earth}/r_{sat}^3 = \omega^2$ so we can solve for $\omega$:

$\omega = [6.67 \times 10^{-11}\ \text{Nt-m}^2/\text{kg}^2 * 6.0 \times 10^{24}\ \text{kg} / (6.72 \times 10^6\ \text{m})^3\ ]^{1/2} =$ **$1.148 \times 10^{-3}$ rad/sec** .

From $\omega = 2\pi f = 2\pi/T$, we can solve for T

$T = 2\pi/\omega = 2 * 3.14 / 1.148 \times 10^{-3}\ \text{sec} = 5{,}470\ \text{sec}$

= **91 minutes**.

From $v_\theta = \omega r$ we get $\mathbf{v_\theta} =$

$1.148 \times 10^{-3}\ \text{rad/sec} * 6.72 \times 10^6\ \text{m} =$ **7,720 m/s** = 17,000 mph

# In general

- In general we can determine the mass of a planet (such as the earth) by watching the orbit of a moon or satellite around the planet - knowing T and r. This applies to the sun as well, since the earth (and other planets) orbit it.
- We can determine the acceleration due to gravity ($g_{planet}$) on a planet's surface by knowing the planet's mass and radius.

# Computer Homework

The computer homework assignment on Circular Motion & Satellites, Volume 1 #7, has problems in this area.

# Rotational Force (Torque)

- Forces cause change in the motion, but so far we have only considered motion that changes the position of the object.
- What about changing the "spin" or rotation of an object?
- To get a nice introduction to the idea of torque, see the **computer homework program on Introduction to Torque (Vol 2 #4)**. This program is due after the test, but you may wish to view it before the test.

# Torque

- There are two important quantities in torque:  Force (F)  and where you apply the force (called  radius, r).

$$\boldsymbol{\tau = r \times F = r\,F\,\sin(\theta_{rF})}$$

To get a large torque, we need to use a large radius, a large force, and apply the force perpendicular to the radius!

# Statics and Torque

Just as $\Sigma\,\mathbf{F} = \mathbf{0}$ when the object is static, so also $\Sigma\,\tau = \mathbf{0}$ when the object is not spinning (or spinning at a constant rate).

In static cases, there is no obvious center to measure the radius from, so we are free to choose any point. However, some points may be simpler to use than others.

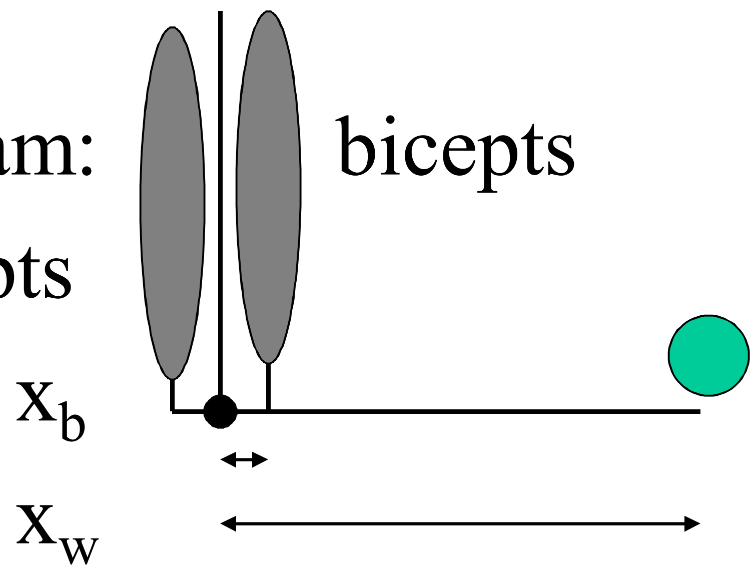
# Your elbow

Let's consider as an example of torque how your muscles, bones and joints work.

Consider holding up a ball of weight 5 lb.

How does this work?

First we draw a diagram:



# Your elbow

- In terms of forces and distances, the diagram looks like this:

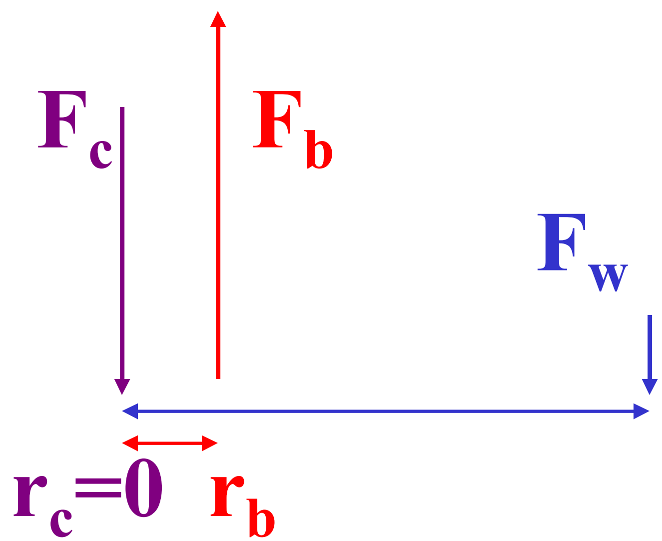
Estimate the distance

from your elbow joint

to your bicept connect

point, $r_b$, and to

your hand, $r_w$.




$r_w$

# Your elbow

- If the ball weights 5 lb, how much force does your biceps pull up with? How much force of contact does your upper arm push down with on your lower arm at the elbow?
- What is the basic principle to use? Statics:

$$\boldsymbol{\Sigma\, F = 0} \quad \text{and} \quad \Sigma\, \tau = 0.$$

# Your elbow

From $\Sigma F = 0$ we have:

$$-\mathbf{F_c} + \mathbf{F_b} - \mathbf{W} = \mathbf{0}$$

And from $\Sigma \tau = 0$ and measuring from the elbow gives: $\mathbf{F_c * r_c + F_b * r_b - W * r_w = 0}$ .

We have two equations and we have two unknowns ($F_c$ and $F_b$).

# Your elbow

- We can use the torque equation first, since $r_c=0$ eliminates one of the unknowns, $F_c$.

$\mathbf{F_c*r_c + F_b*r_b - W*r_w = 0}$ or $F_b = W*r_w/r_b$ .

Then we can use the force equation to find $F_c$ .

$\mathbf{-F_c + F_b - W = 0}$, or $F_c = F_b - W$.

# Your elbow

- By putting in reasonable values for $r_b$ and $r_w$, you can see that the biceps have to excert a large force to hold up a relatively light weight!
- What advantage does this give? Note how far the biceps have to contract in order to move the weight! This is the advantage of the elbow set-up!
- In practice, we use clubs and rackets to make this difference even greater!