Date of Examination 27th November 2011
Time 9.30 to 11.30 Hrs

## Instruction to candidates

1. In addition to this question paper, you are given answer sheet for part $A$ and an answer paper for part B.
2. On the answer sheet for part $A$, fill up all the entries carefully in the space provided, ONLY In BLOCK CAPITALS.
Incomplete / incorrect / carelessly filled information may disqualify your candidature.
3. On part A answer sheet, use only BLUE of BACK BALL PEN for making entries and marking answer.
4. Part A has two parts. In part A1 (Q.No. 1 to 10) each question has Four alternatives, our of which only one is correct. Choose the correct alternative and mark a cross $(X)$ in the corresponding box on the answer sheet.

For example,

| Q. | a | b | $c$ | $d$ |
| :---: | :---: | :---: | :---: | :---: |
| 22 |  |  |  |  |

Part A2 (Q.Nos. 41 to 50) has four alternatives in each question, but any number of these (4, 3, 2, or 1) may be correct. You have to mark ALL correct alternatives and mark a cross (X) for each, like

| Q. | a | b | c | d |
| :---: | :---: | :---: | :---: | :---: |
| 44 |  |  |  |  |

5. For Part A1, each correct answer gets 3 marks; wrong answer gets a penalty of 1 mark. For Part A2 full marks are 6 for each question, you get them when ALL correct answer only are marked.
6. Any rough work should be done only on the sheets provided with part B answer paper.
7. Use of nonprogrammable calculator is allowed.
8. No candidate should leave the examination hall before the completion of the examination.
9. After submitting your answer papers, read the instructions regarding evaluation given at the end of the question paper.

## PLEASE DO NOT MAKE ANY MARKS OTHER THAN (X) IN THE SPACE PROIDED ON THE ANSWER SHEET OF PART A.

Answer sheets are evaluated with the help of a machine. Hence, CHANGE OF ENTRY IS NOT ALLOWED.
Scratching or overwriting may result in wrong score.
DO NOT WRITE ANYTHING ON THE BACK SIDE OF PART A ANSWER SHEET.

## SUB-PART A-1 : ONLY ONE OUT OF FOUR OPTIONS IN CORRECT

## N.B. : Physical constants are given at the end.

## Sub-Part A-1

1. A piece of $n$-type semiconductor is subjected to an electric filed $\mathrm{E}_{\times}$. The left end of the semiconductor is exposed to a radiation so that electron-hole pairs are generated continuously. Let $n$ be the number density of
electrons. The electron current density $J_{e}$ is given by $J_{e}=e n \mu_{e} E_{x}+e D_{e} \frac{d n}{d x}$. The dimension of electron drift mobility ( $\mu_{e}$ ) and electron diffusion coefficient ( $\mathrm{D}_{\mathrm{e}}$ ) are respectively
(a) $\left[\mathrm{M}^{-1} \mathrm{~T}^{-2} \mathrm{I}^{1}\right]$ and $\left[\mathrm{L}^{2} \mathrm{~T}^{-1}\right]$
(b) $\left[\mathrm{M}^{1} \mathrm{~T}^{-2} \mathrm{I}^{-1}\right]$ and $\left[\mathrm{M}^{1} \mathrm{~L}^{2} \mathrm{~T}^{-1}\right]$
(c) $\left[\mathrm{M}^{-1} \mathrm{~T}^{2} \mathrm{I}^{1}\right]$ and $\left[\mathrm{L}^{2} \mathrm{~T}^{-1}\right]$
(d) $\left[\mathrm{M}^{-1} \mathrm{~T}^{2} \mathrm{I}^{2}\right]$ and $\left[\mathrm{L}^{1} \mathrm{~T}^{-2} \mathrm{I}^{1}\right]$

Ans. (c)
Sol. $J=e n \mu_{e} E_{x}+e D_{e} \frac{d n}{d x}$
$\mathrm{IL}^{-2}=\mathrm{MLT}^{-2} \mathrm{~L}^{-3} \mu_{\mathrm{e}}$
$\mu_{\mathrm{e}}=\mathrm{M}^{-1} \mathrm{IT}^{+2}$
$\mathrm{IL}^{-2}=I T L^{-4} \mathrm{D}_{\mathrm{e}}$
$D_{e}=L^{2} T^{-1}$
Aliter : unit of $\frac{\mathrm{I}}{\mathrm{AeE}}=$ unit of $\mu \mathrm{e}$
neAVd
AeE
$\frac{V_{d} \cdot q}{F}$
$\frac{\text { meter }}{\text { sec }} \cdot \frac{\mathrm{q}(\mathrm{sec})^{2}}{\text { mass.meter }}=$ unit of $\mu \mathrm{e}$
$\mathrm{I}^{1} \mathrm{~m}^{-1} \mathrm{~T}^{2} \quad$ Ans. (c)
2. A metal sample carrying a current along $X$ axis with density $J_{x}$ is subjected to a magentic field $B_{z}$ (along $Z$ axis). The electric field $E_{y}$ (Hall field) developed along $Y$ axis is directly proportional to $J_{x}$ as well as $B_{z}$. The constant to proportionality (Hall coefficient) has SI unit
(a) $\mathrm{C} / \mathrm{m}^{2}$
(b) $\mathrm{m}^{2} \mathrm{~s} / \mathrm{C}$
(c) $\mathrm{m}^{2} / \mathrm{C}$
(d) $\mathrm{m}^{3} / \mathrm{C}$

Ans. (d)
Sol. $\quad E_{y}=\mathrm{CJ}_{x} \mathrm{~B}_{z}$
$C=\frac{E_{y}}{J_{x} B_{z}}=\frac{\mathrm{LT}^{-1}}{I L^{-2}}=\mathrm{m}^{3} / \mathrm{C}$
3. A vibratory motion is represented by
$x=2 A \cos \omega t+A \cos \left(\omega t+\frac{\pi}{2}\right)+A \cos (\omega t+\pi)+\frac{A}{2} \cos \left(\omega t+\frac{3 \pi}{2}\right)$. The resultant amplitude of the motion is
(a) $\frac{9 \mathrm{~A}}{2}$
(b) $\frac{\sqrt{5} \mathrm{~A}}{2}$
(c) $\frac{5 \mathrm{~A}}{2}$
(d) 2 A

Ans. (b)

Sol.

$=\sqrt{(A)^{2}+(A / 2)^{2}}$
$=\frac{\sqrt{5} \mathrm{~A}}{2}$
4. A force ( $F$ ) acting on a body is dependent on its displacement $s$ as $F \propto s^{-1 / 3}$ Therefore, the power delivered by the force varies with its displacement as
(a) $\mathrm{s}^{2 / 3}$
(b) $\mathrm{s}^{1 / 2}$
(c) $\mathrm{s}^{-5 / 3}$
(d) $\mathrm{S}^{0}$

Ans. (d)
Sol. $\quad F \propto S^{-1 / 3}$
$\alpha \propto S^{-1 / 3}$
$V^{2} \propto S^{2 / 3}$
$V \propto S^{1 / 3}$
$P=F . V$
$P=F . V \propto S^{-1 / 3} . S^{1 / 3} \propto S^{0}$
5. Bamboo strips are hinged to from three rhombi as shown. Point $A_{0}$ is fixed to a rigid support. The lengths of the sides of the rhombi are in the ratio $3: 2: 1$. Point $A_{3}$ is pulled with a speed $v$. Let $v_{\mathrm{Al}}$ and $v_{\mathrm{A} 2}$ be the speeds with which points $A_{1}$ and $A_{2}$ move. Then, the ratio $v_{A I}: v_{A 2}$ is :

(a) $2: 3$
(b) $3: 5$
(c) $3: 2$
(d) $5: 3$

Ans. (b)

Sol.

$V=\frac{d}{d t}[12 \cos \theta]=-12 \sin \theta \frac{d \theta}{d t}$
$A_{A_{2}}=\frac{d}{d t}[10 \cos \theta]=-10 \sin \theta \frac{d \theta}{d t}$
$A_{A_{1}}=\frac{d}{d t}[6 \cos \theta]=-6 \sin \theta \frac{d \theta}{d t}$
$\frac{\mathrm{V}_{\mathrm{A}_{1}}}{\mathrm{~V}_{\mathrm{A}_{2}}}=\frac{3}{5}$
6. A particle of mass $m$ is made to move with uniform speed $v$ along the perimeter of a regular hexagon. Magnitude of impulse applied at each corner of the hexagon is
(a) $m v$
(b) $m v \sqrt{3}$
(c) $\frac{m v}{2}$
(d) zero

Ans. (a)
Sol. $\quad P_{i}=m v \hat{i}$
$P_{f}=m v \frac{\sqrt{3}}{2} \hat{j}+\frac{m v}{2} \hat{i}$
Impulse $=P_{f}-P_{i}$
$=\frac{\sqrt{3}}{2} m v \hat{j}-\frac{m v}{2} \hat{i}$
$=\sqrt{\left(\frac{\sqrt{3}}{2} m v\right)^{2}+\left(\frac{m v}{2}\right)^{2}}$
$=\mathrm{mv}$
7. Two chambers of different volumes, one containing $m_{1} g$ of a gas at pressure $p_{1}$ and other containing $m_{2} g$ of the same gas at pressure $p_{2}$ are joined to each other. If the temperature of the gas remains constant, the common pressure reached is
(a) $\frac{m_{1} p_{1}+m_{2} p_{2}}{m_{1}+m_{2}}$
(b) $\frac{m_{1} p_{2}+m_{2} p_{1}}{m_{1}+m_{2}}$
(c) $\frac{m_{1} p_{2}\left(p_{1}+p_{2}\right)}{m_{1}^{2}+m_{2}}$
(d) $\frac{\left(m_{1}+m_{2}\right) p_{1} p_{2}}{m_{1} p_{2}+m_{2} p_{1}}$

Ans. (d)
Sol. $\quad P_{1} V_{1}=\frac{m_{1}}{M} R T$
$P_{2} V_{2}=\frac{m_{2}}{M} R T$
$P\left(V_{1}+V_{2}\right)=\left[\frac{m_{1}}{M}+\frac{m_{2}}{M}\right] R T$
$P=\frac{\left(m_{1}+m_{2}\right) P_{1} P_{2}}{m_{1} P_{2}+m_{2} P_{1}}$
8. Two liquid drops of equal radii are falling through air with the terminal velocity $v$. If these two drops coalesce to form a single drop, its terminal velocity will be :
(a) $\sqrt{2 v}$
(b) $2 v$
(c) $\sqrt[3]{4 v}$
(d) $\sqrt[3]{2 v}$

Ans. (c)
Sol. $\quad \mathrm{mg}=6 \pi \eta r v$

$$
\begin{aligned}
2 m g & =6 \pi \eta \cdot 2^{1 / 3} r v^{\prime} \\
v^{\prime} & =2^{2 / 3} v=4^{1 / 3} v=\sqrt[3]{4} v
\end{aligned}
$$

9. An elevator of mass $M$ is accelerated upwards by applying a force $F$. A mass $m$ initially situated at a height of 1 m above the floor of the elevator is falling freely. It will hit the floor of the elevator after a time equal to :
(a) $\sqrt{\frac{2 M}{F+m g}}$
(b) $\sqrt{\frac{2 M}{F-m g}}$
(c) $\sqrt{\frac{2 M}{F}}$
(d) $\sqrt{\frac{2 M}{F+M g}}$

## Ans. (c)

Sol. : acceleration of elevator $=\frac{F-m g}{M}$
acceleration man w.r. to elevator $=g+\frac{F-m g}{M}=\frac{F}{M}$
$\left(S=u t+\frac{1}{2} a t^{2}\right)$ w.r. to elevator
$1=\frac{1}{2} \times \frac{F}{M} \times t^{2}$
$t=\sqrt{\frac{2 M}{F}}$
10. The formation of solid argon is due to van der Waals bonding. In this case the potential energy as a function of interatomic separation can be written as (Lennard Jones 6-12 potential energy) $E(r)=-\mathrm{Ar}^{-6}+\mathrm{Br}^{-12}$ where $A$ and $B$ are constant. Given that $A=8.0 \times 10^{77} \mathrm{Jm}^{6}$ and $B=1.12 \times 10^{-133} \mathrm{Jm}^{12}$, the bond length for solid argon is :
(a) 3.75 nm
(b) 0.0375 nm
(c) 0.750 nm
(d) 0.375 nm

Ans. (d)
Sol. $E(r)=-A r^{-6}+B r^{-12}$
$F=\frac{-d E}{d r}=0$
$6 \mathrm{Ar}^{-7}-12 \mathrm{Br}^{-13}=0$

$$
\begin{aligned}
& r^{6}=\frac{2 B}{A} \\
& r=\left(\frac{2 B}{A}\right)^{1 / 6}=\frac{2 \times 1.12 \times 10^{-56}}{8}=3.75 \times 10^{-10}=0.375 \mathrm{~nm}
\end{aligned}
$$

11. Let $A$ and $B$ be the points respectively above and below the earth's surface each at a distance equal to half the radius of the earth. If the acceleration due to gravity at these points be $g_{A}$ and $g_{B}$ respectively, then $g_{B}: g_{A}$ is :
(a) $1: 1$
(b) $9: 8$
(c) $8: 9$
(d) zero

Ans. (b)
Sol. $\quad \frac{g_{B}}{g_{A}}=\frac{\frac{G R}{R^{3}} \frac{R}{2}}{\frac{4 G M}{9 R^{2}}}=\frac{9}{8}$
12. Let $v_{\mathrm{rms}}, v$ and $v_{\text {avg }}$ represent the root mean square, the most probable and the average velocities respectively, in case of a gaseous system in equilibrium at certain temperature. Then, $v_{\mathrm{rms}}: v: v_{\text {avg }}$ is :
(a) $8: 3 \pi: 2 \pi$
(b) $8: 2 \pi: 3 \pi$
(c) $3 \pi: 2 \pi: 8$
(d) $3: 2: 8$

Ans. (c)
Sol. $v_{r m s}: v: v_{\text {avg }}=\sqrt{\frac{3 R T}{M}}: \sqrt{\frac{2 R T}{M}}: \sqrt{\frac{8 R T}{M}}$
$\mathrm{a} \sqrt{3}: \sqrt{2}: \sqrt{8 / \pi}$
$\sqrt{3 \pi}: \sqrt{2 \pi}: \sqrt{8}$
Aliter. $V_{r m s}=\sqrt{\frac{3 R T}{M}}$
$V_{m \cdot p}=\sqrt{\frac{2 R T}{M}}$
$\mathrm{V}_{\mathrm{ave}}=\sqrt{\frac{8 \mathrm{RT}}{\pi \mathrm{M}}}$
$\mathrm{V}_{\mathrm{rms}}: \mathrm{V}: \mathrm{V}_{\mathrm{ave}}=\sqrt{3 \pi}: \sqrt{2 \pi}: \sqrt{8}$
13. In the arrangement of resistances shown below, the effective resistance between points $A$ and $B$ is :

(a) 23.5 ohm
(b) 38.0
(c) 19.0 ohm
(d) 25.0 ohm

Ans. (c)
Sol. By folding


Balance wheat shone bridge

$R_{A B}=19 \Omega$
14. Ablock of material of specific gravity 0.4 is held submerged at a depth of 1 m in a vessel filled with water. The vessel filled with water. The vessel is accelerated upwards with acceleration of $a_{0}=\frac{g}{5}$. If the block is released at $\mathrm{t}=0$, neglecting viscous effects, it will reach the water surface at t equal to $\left(\mathrm{g}=10 \mathrm{~ms}^{-2}\right)$.
(a) 0.60 s
(b) 0.33 s
(c) 3.3 s
(d) 1.2 s

Ans. (b)

Sol.

$g_{\text {eff }}=12 \quad \frac{\rho_{m}}{\rho_{\mathrm{w}}}=\frac{4}{10}$
$\mathrm{a}=\frac{\mathrm{V} \rho_{\mathrm{w}} \times 12-\mathrm{V} \rho_{\mathrm{m}} \times 12}{\mathrm{~V} . \rho_{\mathrm{m}}}$
$=18 \mathrm{~m} / \mathrm{sec}^{2}$
$1=\frac{1}{2} \times 18 \times t^{2} \quad \Rightarrow \quad t=\frac{1}{3} \mathrm{sec}$
15. The maximum tension in the string of a simple pendulum is 1.2 times the minimum tension. If $\theta_{0}$ is the angular amplitude, then $\theta_{0}$ is :
(a) $\cos ^{-1}\left(\frac{4}{5}\right)$
(b) $\cos ^{-1}\left(\frac{3}{4}\right)$
(c) $\cos ^{-1}\left(\frac{15}{16}\right)$
(d) $\cos ^{-1}\left(\frac{7}{8}\right)$

Ans. (c)

Sol.

$\mathrm{T}=\mathrm{mg}+\frac{\mathrm{mu}^{2}}{\ell}$
$\mathrm{T}^{\prime}=\mathrm{mg} \cos \theta_{0}$
$\frac{m u^{2}}{2}=m g \ell\left(1-\cos \theta_{0}\right)$
$\mathrm{T}=\mathrm{mg}+2 \mathrm{mg}\left(1-\cos \theta_{0}\right)=3 \mathrm{mg}-2 \mathrm{mg} \cos \theta_{0}$
$\mathrm{T}^{\prime}=\mathrm{mg} \cos \theta_{0} \quad \frac{\mathrm{~T}^{\prime}}{\mathrm{T}}=\frac{5}{6}=\frac{\cos \theta_{0}}{3-2 \cos \theta} \Rightarrow \cos \theta_{0}=\frac{15}{16}$
16. A uniform line charge with density $\lambda=50 \mu \mathrm{C} / \mathrm{m}$ lies along X axis. The electric flux per unit length crossing the portion of the plane $z=-3 m$ bounded by $y= \pm 2 m$.
(a) $4.68 \mu \mathrm{C} / \mathrm{m}$.
(b) $9.36 \mu \mathrm{C} / \mathrm{m}$.
(c) $50 \mu \mathrm{C} / \mathrm{m}$.
(d) $18.7 \mu \mathrm{C} / \mathrm{m}$.

Ans. (wrong)

Sol.


$$
\frac{2 \mathrm{k} \lambda}{\sqrt{3^{2}+\mathrm{x}^{2}}} \ell \mathrm{dx}
$$

$\phi=\int_{-2}^{2} \frac{2 \mathrm{k} \lambda}{3^{2}+\mathrm{x}^{2}} \int_{-2}^{2} \frac{2 \mathrm{k} \lambda}{3^{2}+\mathrm{x}^{2}} \ell \mathrm{dx} \quad \Rightarrow \frac{\phi}{\mathrm{Q}}=12 \mathrm{k} \lambda \tan ^{-1}\left(\frac{2}{3}\right)$
$12 \times 10^{-9} \times 50$
17. A plane mirror perpendicular to $X Y$ plane makes an angle of $30^{\circ}$ with the $X$ axis. An object placed at ( $-20,0$ ). Forms an image in the mirror. The point of incidence is $(0,0)$ and the plane of incidence is the XY plane. The coordinates of the image are-
(a) $(10 \sqrt{3}, 10)$
(b) $(-10 \sqrt{3}, 10)$
(c) $(-10,-10 \sqrt{3})$
(d) $(-10 \sqrt{3},-10)$

Ans. (c)

Sol.

18. Magnetic flux through a stationary loop with a resistance $R$ varies during the time interval $\tau$ as $\phi=$ at $(\tau-t)$ where $a$ is a constant. The amount of heat generated in the loop during the time internval $\tau$ is -
(a) $\frac{a^{2} \tau^{3}}{6 R}$
(b) $\frac{a^{2} \tau^{3}}{4 R}$
(c) $\frac{a^{2} \tau^{3}}{3 R}$
(d) $\frac{a^{2} \tau^{3}}{2 R}$

Ans. (c)
Sol. $\quad \phi=$ at $\tau-$ at $^{2}$
$e=\frac{d \phi}{d t}=a \tau-2 a t$
$P=(a \tau-2 a t)^{2} / R$
$H=\int_{0}^{\tau} P d t$
$H=\frac{1}{R}\left[-\frac{(a \tau-2 a t)^{3}}{6 a}\right]_{0}^{\tau}=\frac{a^{2} \tau^{3}}{3 R}$
19. Four functions given below describe motion of a particle. (I) $y=\sin \omega t-\cos \omega t$, (II) $y=\sin ^{3} \omega t$, (III) $y=5$ $\cos \left(\frac{3 \pi}{4}-3 \omega t\right)$, (IV) $y=1+\omega t+\omega^{2} t^{2}$. Therefore, simple harmonic motion is represented by
(a) only (I)
(b) (I), (II) and (III)
(c) (I) and (III)
(d) (I) and (II)

Ans. (c)

Sol. I, III
20. A magnetic field is established with the help of a pair of north and south poles as shown. A small bar magnet placed freely in the field will undergo

(a) pure translational motion
(b) pure rotational motion
(c) rotational motion superimposed on translational motion
(d) oscillatory motion

Ans. (c)
Sol. Due to nonuniform magnetic field force as well as torque will act.
21. In a hydrogen atom, the magnetic field at the center of the atom produced by an electron in the $\mathrm{n}^{\text {th }}$ orbit is proportional to
(a) $\frac{1}{\mathrm{n}^{2}}$
(b) $\frac{1}{\mathrm{n}^{3}}$
(c) $\frac{1}{\mathrm{n}^{4}}$
(d) $\frac{1}{\mathrm{n}^{5}}$

Ans. (d)
Sol. $B=\frac{\mu_{0}}{4 \pi} \frac{q v}{r^{2}}$
$V \propto \frac{1}{n}$
$r \propto n^{2}$
$\mathrm{B} \propto \frac{1}{\mathrm{n}^{5}}$
22. A particle of mass $m$ carries a charge $+q$. It enters into a region of uniform magnetic field $\vec{B}$ existing below the line $\ell \ell^{\prime}$ as as shown. The time spent by the particle in the magnetic field is

(a) $(\pi-2 \theta) \frac{m}{q B}$
(b) infinite as the particle gets trapped
(c) $2 \theta \frac{\mathrm{~m}}{\mathrm{qB}}$
(d) $(\pi+2 \theta) \frac{m}{q B}$

Ans. (d)

Sol. $r=\frac{m v}{q B}$
$\omega=\frac{q B}{m}$
$(\pi+2 \theta)=\omega t$
$\mathrm{t}=\frac{(\pi+2 \theta) \mathrm{m}}{\mathrm{qB}}$

23. $\mathrm{A} 2 \mu \mathrm{~F}$ capacitor is charged as shown in the figure. The charge in its stored energy after the switch S is turned to position 2 is :

(a) $96 \%$
(b) $20 \%$
(c) $4 \%$
(d) $80 \%$

Ans. (a)
Sol. $E=\frac{1}{2} 2 v^{2}$
$Q=2 V$
$\frac{Q_{1}}{2}=\frac{Q_{2}}{8}$
$Q_{1}=\frac{Q_{2}}{4}$
$Q_{1}+Q_{2}=2 V$
$5 Q_{1}=2 V$
$Q_{1}=\frac{2}{5} V$

percentage change in stored energy $=\frac{\frac{1}{2} 2 \mathrm{~V}^{2}-\frac{1}{2} \frac{4}{25} \frac{\mathrm{~V}^{2}}{2}}{\frac{1}{2} 2 \mathrm{~V}^{2}} \times 100$
$=\frac{24}{25} \times 100=96 \%$

## Extra

$2 \mathrm{~V}=2 \mathrm{~V}_{\mathrm{C}}+8 \mathrm{~V}_{\mathrm{C}}$
$2 \mathrm{~V}=10 \mathrm{~V}_{\mathrm{c}}$
$V_{C}=\frac{V}{5}$
$\frac{1}{2} \times(2) \frac{\mathrm{V}^{2}}{25}$
$V=\frac{\mathrm{V}^{2}}{25}$
$\left(\frac{24 \mathrm{~V}^{2}}{25}\right) \times 100$
24. An infinite number of charges each equal to $0.2 \mu \mathrm{C}$ are arranged in a line at distances $1,2,4,8 \ldots \ldots$. meter from a fixed point. The potential at the fixed point is :
(a) 1800 volt
(b) 2000 volt
(c) 3600 volt
(d) 2250 volt

Ans. (c)
Sol. $V=\frac{K \times 0.2 \times 10^{-6}}{1}\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right]$
$=9 \times 10^{9} \times \frac{2}{10} \times 10^{-6} \times \frac{1}{1-\frac{1}{2}}$
$=3600$ Volt.
25. A ball of mass moving with a speed $u$ along a direction making an angle $\theta$ with the vertical strikes a horizontal steel plate. The collision lasts for a time interval $t$. If e is the coefficient of restitution between the ball and the plate, the average force exerted by the plate on the ball is :
(a) $\frac{e m u}{t}$
(b) $\frac{e m u \cos \theta}{t}$
(c) $\frac{2(e+1) m u \cos \theta}{t}$
(d) $\frac{(1+e) m u \cos \theta}{t}$

Ans. (d)

## 31111111111111111111

$u \cos \theta$
Sol.

$F=\frac{\Delta \mathrm{P}}{\mathrm{t}}=\frac{(\mathrm{e}+1) \mathrm{mu} \cos \theta}{\mathrm{t}}$
26. In a Young's double slit experiment sources of equal intensities are used. Distance between slits is d and wavelength of light used is $\lambda(\lambda \ll \mathrm{d})$. Angular separation of the nearest points on either side of central maximum where intensities become half of the maximum value is :
(a) $\frac{\lambda}{d}$
(b) $\frac{\lambda}{2 d}$
(c) $\frac{\lambda}{4 d}$
(d) $\frac{\lambda}{6 d}$

Ans. (b)
Sol. $2 I=I+I+2 \sqrt{I} \sqrt{I} \cos \theta$
$\theta=\frac{\pi}{2}$
$\Delta x=\frac{\lambda}{4}$
$\theta=\frac{\lambda}{4 d}$
Total angular separation $=2 \theta=\frac{\lambda}{2}$
27. The variation of magnetic field along the axis of a solenoid is graphically represented by ( $O$ is the centre with I, l' as the extremities of the solenoid along the axis)
(a)

(b)

(c)

(d)


Ans. (d)
28. A wooden cube is placed on a rough horizontal table. A force is applied to the cube. Gradually the force is increased. Whether the cube slides before toppling or topples before sliding is independent of :
(a) the position of point of application of the force.
(b) the length of the edge of the cube.
(c) mass of the cube
(d) coefficient of friction between the cube and the table.

Ans. (c)
Sol. For toppling $m g \frac{\ell}{2}=F_{1} \times h$
For sliding $\mu \mathrm{mg}=\mathrm{F}_{2}$
Condition for sliding to occurs first $F_{1}>F_{2}$
$\frac{\mathrm{mg} \ell}{2 \mathrm{~h}}>\mu \mathrm{mg}$

$\ell>\mu \mathrm{h}$
29. There are two organ pipes of the same length and the same material but of different radii. When they are emitting fundamental notes.
(a) broader pipe gives note of smaller frequency
(b) both the pipes give notes of the same frequency
(c) narrower pipe gives note of smaller frequency
(d) either of them gives note of smaller or larger frequency depending on the wavelength of the wave.

Ans. (a)

Sol.


$$
f=\frac{v}{\lambda}=\frac{v}{(\ell+e)}
$$

$$
f=\frac{v}{4(L+e)}
$$

$$
(e=0.3 r)
$$

Broader pipe has more value of e therefore it gives note of smaller frequency
30. The wavelength of sodium line observed in the spectrum of a star is found to be 598 nm , whereas that from the sodium lamp in the laboratory is found to be 589 nm . Therefore, the star is moving with a speed of about
(a) $2.7 \times 10^{6} \mathrm{~m} / \mathrm{s}$ away from the earth
(b) $5.4 \times 10^{6} \mathrm{~m} / \mathrm{s}$ towards the earth
(c) $1.6 \times 10^{6} \mathrm{~m} / \mathrm{s}$ away from the earth
(d) $4.6 \times 10^{6} \mathrm{~m} / \mathrm{s}$ away from the earth

Ans. (d)
Sol. $\lambda^{\prime}=\frac{\lambda}{\left(1 \pm \frac{v}{c}\right)}$
$598=\frac{589}{\left(1 \pm \frac{v}{c}\right)} \quad v=3 \times 10^{8} \times 1.5 \times 10^{-2}$
$v=4.52 \times 10^{6} \mathrm{~m} / \mathrm{s}$ away form earth.
31. In a series LCR circuit, impedance $Z$ is the same at two frequencies $f_{1}$ and $f_{2}$. Therefore, the resonant frequency of this circuit is
(a) $\frac{f_{1}+f_{2}}{2}$
(b) $\frac{2 f_{1} f_{2}}{f_{1}+f_{2}}$
(c) $\frac{\sqrt{f_{1}^{2}+f_{2}^{2}}}{2}$
(d) $\sqrt{f_{1} f_{2}}$

Ans. (d)
Sol. $\quad Z=\sqrt{\left(X_{L}-x_{C}\right)^{2}+R^{2}}$
$Z$ is same for two frequencies then ,
$\left(\mathrm{X}_{\mathrm{L}}-\mathrm{x}_{\mathrm{C}}\right)_{\mathrm{f}}=\left(\mathrm{x}_{\mathrm{C}}-\mathrm{x}_{\mathrm{L}}\right) \mathrm{f}_{\mathrm{f}}$
$2 \pi f_{1} L-\frac{1}{2 \pi f_{1} C}=\frac{1}{2 \pi f_{2} C}-2 \pi f_{2} L$
$2 \pi\left(f_{1}+f_{2}\right) L=\frac{1}{2 \pi C}\left(\frac{1}{f_{1}}+\frac{1}{f_{2}}\right)$
$4 \pi^{2}\left(f_{1}+f_{2}\right)=\frac{1}{L C}\left(\frac{1}{f_{1}}+\frac{1}{f_{2}}\right)$
$w_{r}=\frac{1}{\sqrt{\text { LC }}}$
$4 \pi^{2}=\frac{w_{r}^{2}}{f_{1} f_{2}}$
$4 \pi^{2}=\frac{4 \pi^{2} f_{r}^{2}}{f_{1} f_{2}}$
$\mathrm{f}_{\mathrm{r}}{ }^{2}=\mathrm{f}_{1} \mathrm{f}_{2}$
$\mathrm{f}_{\mathrm{r}}=\sqrt{\mathrm{f}_{1} \mathrm{f}_{2}}$
32. Two particles are moving along $X$ and $Y$ axes towards the origin with constant speed $u$ and $v$ respectively. At time $t=0$, their respective distance from the origin are $x$ and $y$. The time instant at which the particles will be closest to each other is
(a) $\frac{\sqrt{x^{2}+y^{2}}}{\sqrt{u^{2}+v^{2}}}$
(b) $\frac{v x+u y}{u^{2}+v^{2}}$
(c) $\frac{u x+v y}{u^{2}+v^{2}}$
(d) $\frac{2 \sqrt{x^{2}+y^{2}}}{u+v}$

Ans. (c)

Sol. Use
$r_{\text {min }}=\frac{\left|\vec{r}_{12} \times \vec{v}_{12}\right|}{\left|\vec{v}_{12}\right|}$
33. A container of volume $0.1 \mathrm{~m}^{3}$ is filled with nitrogen at a temperature of $47^{\circ} \mathrm{C}$ and a gauge pressure of $4.0 \times 10^{5} \mathrm{~Pa}$. After some time, due to leakage, the gauge pressure drops $3.0 \times 10^{5} \mathrm{~Pa}$ and the temperature to $27^{\circ} \mathrm{C}$. The mass of nitrogen that has leaked out is about
(a) 128 g
(b) 84 g
(c) 154 g
(d) 226 g

Ans. most appropriate (b)
$P v=\frac{m R T}{m w}$
$4 m=\frac{p_{1} v m_{w}}{R T_{1}}-\frac{p_{2} v m_{w}}{R T_{2}}$
$=\frac{0.1 \times 28 \times 10^{-3}}{8.31}\left\{\frac{4 \times 10^{5}}{(273+47)}-\frac{3 \times 10^{5}}{300}\right\}$
$=84 \times 10^{-3} \mathrm{~kg}$
$4 \mathrm{~m}=84 \mathrm{gm}$
34. Ninety percent of a radioactive sample is left over after a time interval $t$. The percentage of initial sample that will disintegrate in an interval 2 t is
(a) $38 \%$
(b) $19 \%$
(c) $9 \%$
(d) $62 \%$

Ans. (b)
Sol. $\quad N=N_{e} e^{-\lambda t}$
$\mathrm{t}=\frac{1}{\lambda} \ell \mathrm{n}\left(\frac{\mathrm{N}_{0}}{0.9 \mathrm{~N}}\right)$
$2 \mathrm{t}=\frac{1}{\lambda} \ell \mathrm{n}\left(\frac{\mathrm{N}_{0}}{\mathrm{~N}}\right)$
$2 \frac{1}{\lambda} \ell n\left(\frac{N_{0}}{0.9 \mathrm{~N}_{0}}\right)=\frac{1}{\lambda} \ell n\left(\frac{\mathrm{~N}_{0}}{\mathrm{~N}}\right)$
$\frac{\mathrm{N}_{0}^{2}}{0.81 \mathrm{~N}_{0}^{2}}=\frac{\mathrm{N}_{0}}{\mathrm{~N}}$
$\mathrm{N}=0.81 \mathrm{~N}_{0}$
The percentage of initial sample that is disintegrated $=19 \%$
35. A circuit is arranged as shown. At time $t=0 \mathrm{~s}$, switch $S$ is placed in position $I$. At $t=5 \mathrm{~s}$, contact is changed from 1 to 2 . The voltage across the capacitor is measured at $t=5 \mathrm{~s}$, and at $\mathrm{t}=6 \mathrm{~s}$. Let these voltages be $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ respectively. Then $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ respectively are

(a) 10 volt and 0 volt
(b) 9.18 volt and 3.67 volt
(c) 9.18 volt and 3.37 volt.
(d) 10 volt and 3.67 volt

Ans. (c)
a) $\quad V=V_{0}\left(1-e^{-\frac{t}{2 R C}}\right)$

$$
\begin{aligned}
V & =V_{0}\left(1-e^{-\frac{5}{2}}\right) \\
& =9.18 \text { volt }
\end{aligned}
$$

(b) $\quad V=V_{0}\left(e^{-\frac{t}{R C}}\right)$

$$
\begin{aligned}
& =9.18 \mathrm{e}^{-1 / 1} \\
& =\frac{9.18}{\mathrm{e}}=3.37 \mathrm{volt}
\end{aligned}
$$

36. There are two thermocouples $A$ and $B$ made of the same pair of metals. In $A$ each wire is 50 cm long and in $B$ each wire is 150 cm long. Both the thermocouples are maintained between the same lower temperature $\theta_{1}$ and higher temperature $\theta_{2}$. Each of the thermocouples is connected to the same microammeter successively. Let $\varepsilon$ be the thermoemf and I be the thermoelectric current. Then which of the following statements is true ?
(a) Both $\varepsilon$ and $I$ are equal for $A$ and $B$
(b) Both $\varepsilon$ and I are greater for $B$ than those for $A$
(c) $\varepsilon$ is the same for both but $I$ is greater for $A$
(d) $\varepsilon$ is the same for both but $I$ is greater for $B$

Ans. (c)
37. Two identical particles move at right angles to each other, processing de Broglie wavelengths $\lambda_{1}$ and $\lambda_{2}$. The de Broglie wavelength of each of the particles in their centre of mass frame will be
(a) $\sqrt{\frac{\lambda_{1}^{2}+\lambda_{2}^{2}}{2}}$
(b) $\frac{\lambda_{1}+\lambda_{2}}{2}$
(c) $\frac{2 \lambda_{1} \lambda_{2}}{\lambda_{1}+\lambda_{2}}$
(d) $\frac{2 \lambda_{1} \lambda_{2}}{\sqrt{\lambda_{1}^{2}+\lambda_{2}^{2}}}$

Ans. (d)
Sol. $\lambda=\frac{\mathrm{h}}{\mathrm{mv}_{1}}, \lambda_{2}=\frac{\mathrm{h}}{\mathrm{mv}_{2}}$
The de Broglie wavelength of one of the particles in their centre of mass frame is

$$
\begin{gathered}
\lambda=\frac{h}{m \times v_{\text {rel }}}=\frac{h}{m \sqrt{\left(\frac{v_{1}}{2}\right)^{2}+\left(\frac{v_{2}}{2}\right)^{2}}}=\frac{2 h}{m \sqrt{\left(\frac{h}{m \lambda_{1}}\right)^{2}+\left(\frac{h}{m \lambda_{2}}\right)^{2}}} \\
=\frac{2 \lambda_{1} \lambda_{2}}{\sqrt{\lambda_{1}^{2}+\lambda_{2}^{2}}}
\end{gathered}
$$

38. The stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength 400 nm is 500 mV . When the incident wavelength is changed to a new value, the stopping potential is found to be 800 mV . New wavelength is about
(a) 365 nm
(b) 250 nm
(c) 640 nm
(d) 340 nm

Ans. (a)
$\mathrm{I}_{1}=400 \mathrm{~nm}$
$\mathrm{v}_{1}=500 \mathrm{mV}$
$\mathrm{v}_{2}=800 \mathrm{mV}$
$\lambda_{2}=$ ?
$e V_{1}=\frac{h c}{\lambda_{1}}-\phi$
$\mathrm{eV}_{2}=\frac{\mathrm{hc}}{\lambda_{2}}-\phi$

By solving eqn (1) and (2)
$\lambda_{2}=365 \mathrm{~nm}$
39. For the logic circuit given below, the outputs $Y$ for $A=0, B=0$ and $A=1, B=1$

(a) 0 and 1
(b) 0 and 0
(c) 1 and 0
(d) 1 and 1

Ans. (b)
Sol. When $A=0$ and $B=0$


Ans. $Y=0$
When $A=1$ and $B=1$


Ans. $Y=0$
40. In a hydrogen like atom electron makes transition from an energy level with quantum number $n$ to another with quantum number $(n-1)$. If $n \gg 1$, the frequency of radiation emitted is proportional to :
(a) $\frac{1}{\mathrm{n}^{2}}$
(b) $\frac{1}{\mathrm{n}^{3}}$
(c) $n^{2}$
(d) $\frac{1}{\mathrm{n}^{4}}$

Ans. (b)
Sol. $E_{n}-E_{n-1}$
$=\frac{-13.6 Z^{2}}{n^{2}}+\frac{-13.6 Z^{2}}{(n-1)^{2}}$
$=13.6 Z^{2}\left[\frac{1}{(n-1)^{2}}-\frac{1}{n^{2}}\right]$
$h v=13.6 Z^{2}\left[\frac{n^{2}-(n-1)^{2}}{n^{2}(n-1)^{2}}\right]$
$=\frac{13.6 Z^{2}}{n^{2}}\left[\frac{1}{(1-1 / n)^{2}}-1\right]$
$=\frac{13.6 Z^{2}}{n^{2}}\left[\left(1-\frac{1}{n}\right)^{-2}-1\right]$
$=\frac{13.6 Z^{2}}{n^{2}}\left[1+\frac{2}{n}-1\right]=\frac{2 \times 13.6 Z^{2}}{n^{3}} \quad v \propto \frac{1}{n^{3}}$
(b)

In question 41 to 50 any number of options (1 or 2 or 3 or all 4) may be correct. You are to identify all of them correctly to get 6 marks. Even if one answer identified is incorrect or one correct answer is missed, you get zero score.
41. Consider an electron orbiting the nucleus with speed $v$ in an orbit of radius $r$. The ratio of the magnetic moment to the orbital angular momentum of the electron is independent of :
(a) radius $r$
(b) speed $v$
(c) charge of electron e
(d) mass of electron $\mathrm{m}_{\mathrm{e}}$

Ans. (a, b)
Sol. $\frac{m v^{2}}{r}=\frac{k(z e) e}{r^{2}}$

$i=\left(\frac{e v}{2 \pi r}\right)$
$\mu=\left(\frac{\mathrm{ev}}{2 \pi \mathrm{r}}\right) \pi \mathrm{r}^{4}=\left(\frac{\mathrm{ev}}{}{ }^{r}\right)$
$\ell=\mathrm{mvr}$
$\Rightarrow \frac{\mu}{\ell}=\frac{e v r}{2 m v r}=\left(\frac{e}{2 m}\right)$
42. A current $I_{0}$ enters into a parallel combination of resistors $R_{1}$ and $R_{2}$. Current $I_{1}$ flows through $R_{1}$ and $I_{2}$ through $\mathrm{R}_{2}$. The current $\mathrm{I}_{0}$ distributes in such a way that
(a) Power consumed in $R_{1}$ and in $R_{2}$ is the same.
(b) total power consumed in $R_{1}$ and $R_{2}$ is minimum.
(c) $I_{1}$ is proportional to $R_{2}$ and $I_{2}$ is proportional to $R_{1}$.
(d) the power consumed in each of $R_{1}$ and $R_{2}$ is minimum.

Ans. (c)

Sol.

$\mathrm{I}_{1} \mathrm{R}_{1}=\mathrm{I}_{2} \mathrm{R}_{2}$
$I_{0}=I_{1}+I_{2}$
$I_{0}=I_{2}\left(\frac{R_{1}+R_{2}}{R_{1}}\right)$
$I_{2}=I_{0}\left(\frac{R_{1}}{R_{1}+R_{2}}\right)$
$I_{1}=I_{0}\left(\frac{R_{2}}{R_{1}+R_{2}}\right)$

$P=x^{2} R_{1}+\left(I_{0}-x\right)^{2} R_{2}$
$\frac{d P}{d x}=2 x R_{1}-2(I-x) R_{2}$
$\frac{d^{2} P}{d x^{2}}=2\left(R_{1}+R_{2}\right)>0$
Therefore minimum power
43. Weight of a body on the surface of the earth depends on :
(a) distance of the body from the centre of the earth
(b) the latitude of the place on the earth surface where the body is placed
(c) the longitude of the place on the earth surface where the body is placed.
(d) the angular speed of rotation of the earth about its own axis.

Ans. (a, b, d)
44. Which of the following is/are involved in the formation of rain drops in a cloud ?
(a) saturation of vapour pressure
(b) temperature
(c) viscosity
(d) surface tension

Ans. (a, d)
45. A cyclic process on P-V diagram is as shown below. The same process can be shown on p-T or V-T diagrams. Choose the correct alternative/s


Ans. (a,b)

## Test - 01


(a)


(b)

(c)
(d)

Sol. $\quad P V=\mu R T$
$p=\left(\frac{\mu R}{v}\right) T$
for, $\mathrm{v}=$ constant
$\Rightarrow \quad \mathrm{p} \propto \mathrm{T}$
ans (a), (b)
and
$p V=\mu R T$
$v=\left(\frac{\mu \mathrm{R}}{\mathrm{P}}\right) \mathrm{T}$
$\mathrm{v}=$ constant
$v \propto T$
46. When a bright light source is placed 30 cm in front of a thin lens, an erect image is formed at 7.5 cm from the lens. A faint inverted image is also formed at 6 cm in front of the lens due to reflection from the front surface of the lens. When the lens is turned around, this weaker inverted image is now formed at 10 cm in front of the lens.
Therefore,
(a) the lens is diverging biconcave
(b) the refractive index of the glass of the lens is 1.6
(c) radii of curvature of surfaces of the lens are 10 cm and 15 cm respectively.
(d) the lens behaves as converging lens of focal length 30 cm when immersed in a liquid of refractive index 2.

Ans. (a, b, c, d)

Sol.

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$
$-\frac{2}{15}+\frac{1}{30}=\frac{1}{f} \Rightarrow \frac{-4}{30}+\frac{1}{30}=\frac{1}{f}$
$f=-\frac{30}{3}=-10 \mathrm{~cm}$

$\frac{1}{v}+\frac{1}{u}=\frac{2}{R_{1}}$
$\frac{5}{-6 \times 5}+\frac{1}{-30}=\frac{2}{R_{1}}$
$-\frac{6}{30}=\frac{2}{R_{1}}$
$R_{1}=-\frac{2 \times 30}{6}=-10 \mathrm{~cm}$
$\frac{1}{-10}+\frac{1}{-30}=\frac{2}{R_{2}}$
$-\frac{3}{30}-\frac{1}{30}=\frac{2}{R_{2}}$
$-\frac{4}{30}=\frac{2}{\mathrm{R}_{2}}$

$$
\mathrm{R}_{2}=\frac{2 \times 30}{-4}=-15 \mathrm{~cm}
$$

$\frac{1}{f}=\left(\frac{\mu_{2}}{\mu_{1}}-1\right)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$
$-\frac{1}{10}=(\mu-1)\left(\frac{1}{-10}-\frac{1}{15}\right)$
$-\frac{1}{10}=(\mu-1)\left(\frac{-3-2}{30}\right)$
$\mu=1.6$
47. Let $\mathrm{n}_{1}$ and $\mathrm{n}_{2}$ moles of two different ideal gases be mixed. If ratio of specific heats of the two gases are $\gamma_{1}$ and $\gamma_{2}$ respectively, then the ratio of specific heats $\gamma$ of the mixture is given through the relation
(a) $\left(\mathrm{n}_{1}+\mathrm{n}_{2}\right) \gamma=\mathrm{n}_{1} \gamma_{1}+\mathrm{n}_{1} \mathrm{r}_{2}$
(b) $\frac{\left(\mathrm{n}_{1}+\mathrm{n}_{2}\right)}{\gamma-1}=\frac{\mathrm{n}_{1}}{\gamma_{1}-1}+\frac{\mathrm{n}_{2}}{\gamma_{2}-1}$
(c) $\left(\mathrm{n}_{1}+\mathrm{n}_{2}\right) \frac{\gamma}{\gamma-1}=\mathrm{n}_{1} \frac{\gamma_{1}}{\gamma-1}+\mathrm{n}_{2} \frac{\gamma_{2}}{\gamma_{2}-1}$
(d) $\left(\mathrm{n}_{1}+\mathrm{n}_{2}\right)(\gamma-1)=\mathrm{n}_{1}\left(\gamma_{1}-1\right)+\mathrm{n}_{2}\left(\gamma_{2}-1\right)$

Ans. (b, c)
Sol. $\quad y_{\text {mix }}=\frac{\left(\frac{n_{1} y_{1}}{y_{1}-1}\right)+\left(\frac{n_{2} y_{2}}{y_{2}-1}\right)}{\left(\frac{n_{1}}{y_{1}-1}\right)+\left(\frac{n_{2}}{y_{2}-1}\right)}=y$
$\frac{n_{1}+n_{2}}{\left(\frac{n_{1}}{y_{1}-1}\right)+\left(\frac{n_{2}}{y_{2}-1}\right)}=(y-1)$
(1) : (2) $\frac{\left(n_{1}+n_{2}\right) y}{y-1}=\frac{n_{1} y_{1}}{y_{1}-1}+\frac{n_{2} y_{2}}{y_{2}-1}$

Aliter $\quad\left(C_{v}\right)_{\text {mix }}=\frac{n_{1}\left(\frac{R}{r_{1}-1}\right)+n_{2}\left(\frac{R}{r_{2}-1}\right)}{n_{1}+n_{2}}$
$\left(C_{p}\right)_{\text {mix }}=\frac{n_{1} v_{1}\left(\frac{R}{r_{1}-1}\right)+n_{2} v_{2}\left(\frac{R}{r_{2}-1}\right)}{\left(n_{1}+n_{2}\right)}$
$V_{\text {mix }}=\frac{\left(C_{p}\right)}{\left(C_{v}\right)}=\frac{n_{1} v_{1}\left(\frac{R}{r_{1}-1}\right)+n_{2} v_{2}\left(\frac{R}{r_{2}-1}\right)}{n_{1}\left(\frac{R}{r_{1}-1}\right)+n_{2}\left(\frac{R}{r_{2}-1}\right)}$
$=\frac{n_{1} v_{1}+n_{2} v_{2}}{\left(n_{1}+v_{1}\right)}$
$V_{\text {mix }}=\frac{n_{1}\left(\frac{r_{1}}{r_{1}-1}\right)+n_{2}\left(\frac{r_{2}}{r_{2}-1}\right)}{n_{1}\left(\frac{r_{1}}{r_{1}-1}\right)+n_{2}\left(\frac{r_{2}}{r_{2}-1}\right)}$
48. A resistance of 4 ohm is connected across a cell. Then it is replaced by another resistance of 1 ohm . It is found that power dissipated in resistance in both the cases is 16 watt. Then,
(a) internal resistance of the cell is 2 ohm.
(b) emf of the cell is 12 volt.
(c) maximum power that can be dissipated in the external resistance is 18 watt.
(b) short circuit current from the cell is infinite.

## Ans. (a,b,c)

Sol. $i=\left(\frac{\varepsilon}{4+r}\right)$

$$
\begin{aligned}
& \mathrm{p}_{1}=\left(\frac{\varepsilon}{4+\mathrm{r}}\right)^{2}(4) \\
& \mathrm{p}_{1}=\left(\frac{\varepsilon}{1+\mathrm{r}}\right)^{2}(1) \\
& \left(\frac{\varepsilon}{4+\mathrm{r}}\right)^{2}(4)=\left(\frac{\varepsilon}{1+\mathrm{r}}\right)^{2}(1) \\
& 2+\mathrm{r} \\
& 2+\frac{\varepsilon}{(1+\mathrm{r})} \\
& \mathrm{r}=2 \\
& 16=\left(\frac{\varepsilon}{6}\right)^{2} \times 4 \\
& 2=\frac{\varepsilon}{6} \\
& \varepsilon=12 \mathrm{~V} \\
& \mathrm{p}_{\max }=\left(\frac{12}{4}\right)^{2} \times 2 \\
& =18 \mathrm{volt}
\end{aligned}
$$

49. Two solid spheres $A$ and $B$ of equal volumes but different densities $d_{A}$ and $d_{B}$ respectively, are connected by a string. They are fully immersed in fluid of density $\mathrm{d}_{\mathrm{F}}$. They get arranged in an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if.

(a) $d_{A}<d_{F}$.
(b) $d_{B}<d_{F}$.
(c) $d_{A}<d_{F}$.
(d) $\left(d_{A}+d_{B}\right)=2 d_{F}$

Ans. (a, b, d)
Sol. $\quad 2 V d_{F}=g=V\left(d_{A}+d_{B}\right) g$
$2 d_{F}=\left(d_{A}+d_{B}\right)$

for ' B '
$T+V d_{F} g=V d_{B} g$
$T+V\left(d_{B}-d_{F}\right) g$
$d_{B}>d_{F}$
for ' A '
$V d_{F} g=T+V d_{A} g$
$T=V\left(d_{F}-d_{A}\right) g, d_{F}>d_{A}$.
50. A particle $Q$ is moving along + $Y$ axis. Another particle $P$ is moving in $X Y$ plane along a straight line $x=-d$ $(d>0)$ with a uniform speed $v$ parallel to that of $Q$. At time $t=0$, particle $P$ and $Q$ happen to be along $X$ axis whereas a third particle $R$ situated at $x=+d$ starts moving opposite to $P$ with a constant acceleration $a$. At all further instants the three particles happen to be collinear. Then, Q
(a) has an initial speed $\frac{v}{2}$
(b) will come to rest after a time interval $\frac{v}{a}$
(c) has an acceleration $-\frac{a}{2}$.
(d) will return to its initial position after a time interval $\frac{2 \mathrm{v}}{\mathrm{a}}$.

Ans. (a, b, c, d)

Sol.

$\frac{v t+\frac{1}{2} a+\ell}{2 d}=\frac{v^{1} t+\frac{1}{2} a+\ell}{d}$
$V+\frac{1}{2} a t=2 V^{1}+a^{1} t=-d$
$0=\frac{V}{L}-\frac{a}{2} \times t$
$\frac{V}{L}-\frac{a}{L} t$
PART B
MARKS : 60

All questions are compulsory.
All questions carry equal marks.

1. (a) A 40 watt, 120 volt incandescent bulb has a tungsten filament 0.381 m long. The diameter of the filament is $33 \mu \mathrm{~m}$. Tungsten has resistivity $5.51 \times 10^{-8}$ ohm-m at room temperature $\left(20^{\circ} \mathrm{C}\right)$ Given that the resistivity of the tungsten filament varies as $T^{6 / 5}$, estimate the temperature of the filament when it is operated at its voltage.
(b) Assume that the electrical power dissipated in the filament is radiated from the surface of the filament. If emissivity of the filament surface is 0.35 , determine the temperature of the filament compare it with that obtained in part (a).
Ans. (a) $2425.53^{\circ} \mathrm{C}$
Sol. $\quad R_{\text {Raled }}=\frac{(120)^{2}}{40}$
$\rho^{\frac{\ell}{\mathrm{A}}}=360 \Omega$
$\rho=\frac{A}{\ell}(360)$
$=\frac{\pi}{4} \times \frac{\left(33 \times 10^{-6}\right)^{2} \times 360}{0.381}$
$=80.77 \times 10^{-8} \Omega-\mathrm{m}$
$\rho^{\prime}=5.51 \times 10^{-8} \Omega-m$
$\frac{\rho}{\rho^{\prime}}=\left(\frac{T}{T^{\prime}}\right)^{\frac{6}{5}}$
$T^{\prime}=\left(\frac{\rho^{\prime}}{\rho}\right)^{\frac{5}{6}} \mathrm{~T}$
$=\left(\frac{80.77 \times 10^{-8}}{5.51 \times 10^{-8}}\right)^{\frac{5}{6}} 293$
$=(14.659)^{5 / 6} 293$
$=(14.659)^{5 / 6} 293-273$
$=(9.21) 293-273$
= 2698-273
$=2425.53^{\circ} \mathrm{C}$
(b) $\frac{d Q}{d t}=\sigma A e\left(T^{4}-T^{4}\right)$
$40=\mathrm{S}(2 \pi \gamma \ell) \mathrm{e}\left(\mathrm{T}^{4}-\mathrm{T}^{4}\right) \quad \mathrm{T}=1125.97 \mathrm{~K}$
$\mathrm{T}=852.97^{\circ} \mathrm{C}$
2. A parallel plate capacitor with a separation $d=1.0 \mathrm{~cm}$ is subjected to a potential difference of 20 kV with air as a dielectric. Assume that air behaves as a dielectric (insulator) upto a maximum electric field (called dielectric strength) of $30 \mathrm{kV} / \mathrm{cm}$ (after which it breaks down). Now a thin plate of glass (dielectric constant $\mathrm{K}=6.5$ and dielectric strength $=290 \mathrm{kV} / \mathrm{cm}$ ) is inserted. Determine the maximum thickness of glass plate to avoid breakdown in the capacitor.

Ans. $\frac{13}{33} \mathrm{~cm}$

Sol.


$$
Q=\frac{\epsilon_{0} A}{\frac{t}{k}+(d-t)} V
$$

$$
E_{\text {air }}=\frac{\epsilon_{0} A V}{\frac{t}{k}+(d-t)} \frac{1}{A \epsilon_{0}}<3 \times 10^{6}
$$

$$
E_{\text {med }}=\frac{\epsilon_{0} A V}{\frac{t}{k}+(d-t)} \frac{1}{k A \varepsilon_{0}}<29 \times 10^{6}
$$

$$
\frac{20000}{\frac{t}{6.5}+10^{-2}-t}<3 \times 10^{6}
$$

$$
\Rightarrow \quad t\left(\frac{1}{6.5}-1\right)+10^{-2}>\frac{2 \times 10^{4}}{3 \times 10^{6}}
$$

$$
\Rightarrow \quad t\left(\frac{1}{6.5}-1\right)+10^{-2}>\frac{2}{3} \times 10^{-2}
$$

$$
\Rightarrow \quad \frac{1}{3} \times 10^{-2}>\frac{5.5}{6.5} t
$$

$$
\Rightarrow \quad \mathrm{t}<\frac{65}{55} \frac{1}{3} \times 10^{-2}
$$

$$
\Rightarrow \quad t<\frac{13}{33} \mathrm{~cm}
$$

Similarly: $\quad \frac{20000}{(t+6.5)\left(10^{-2}-t\right)}<29 \times 10^{6}$

$$
\begin{aligned}
& \mathrm{t}+6.5\left(10^{-2}-\mathrm{t}\right)>\frac{2}{3} \times 10^{-2} \\
& \Rightarrow \quad 5.5 \mathrm{t}<\left(6.5-\frac{2}{3}\right) \mathrm{cm} \\
& \Rightarrow \quad \mathrm{t}<\frac{6.5-2 / 3}{5.5} \mathrm{~cm} \\
& \quad<\frac{35}{33} \mathrm{~cm}
\end{aligned}
$$

Aliter : $\mathrm{q}=\left(\frac{\varepsilon_{0} \mathrm{~A}}{\mathrm{~d}}\right) \mathrm{v}$

$$
\begin{aligned}
& q^{\prime}=\frac{\varepsilon_{0} A}{\left(d-t+\frac{t}{v}\right)} \\
& \frac{q^{\prime}}{A}=\frac{\varepsilon_{0} A}{\left(d-t+\frac{t}{v}\right)} \\
& E=\frac{r}{A q_{0}}=\frac{v}{d-t+\frac{t}{k}} \\
& 30 \times 10^{3}=\frac{20 \times 10^{3}}{1-t+\frac{t}{6.5}} \\
& t=\frac{13}{33} \mathrm{~cm}
\end{aligned}
$$

3. (a) Cauchy's empirical formula for refractive index of a transparent medium is

$$
n-1=A\left(1+\frac{B}{\lambda^{2}}\right)
$$

Hence obtain the condition for achromatic combination of two lenses made from different glasses. Refractive indices of flint glass and crown glass given below.

|  | Red light $\lambda=640 \mathrm{~nm}$ | Blue light $\lambda=480 \mathrm{~nm}$ |
| :---: | :---: | :---: |
| Flint glass | 1.644 | 1.664 |
| Crown glass | 1.514 | 1.524 |

(b) Determine the focal lengths of the two lenses (one of flint glass and the other of crown glass) such that their combination has focal length of +40 cm for all colours.
Ans.
(a) $.035 \mathrm{f}_{1}+.18 \mathrm{f}_{2}=0$
(b) $f_{1}=173.6 \mathrm{~cm}, f_{2}=+32.49 \mathrm{~cm}$

Sol. $-\frac{d f}{f^{2}}=d n\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$
$\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$
$=\frac{\mathrm{df}}{\mathrm{f}}=\left(\frac{\mathrm{dn}}{\mathrm{n}-1}\right)$
$\mathrm{df}_{1}+\mathrm{df}_{2}=0$
$\frac{w_{1}}{f_{1}}+\frac{w_{2}}{f_{2}}=0$
$\frac{\frac{1.664-1.644}{\left(\frac{1.664+.644}{2}-1\right)}}{\mathrm{f}_{1}}+\frac{\frac{1.524-1.514}{\left(\frac{1.524+.514}{2}-1\right)}}{\mathrm{f}_{2}}=0$
$\frac{\frac{.02}{.654}}{f_{1}}+\frac{.01}{f_{2}}=0$
$\frac{.02 \times f_{1}}{.654}+\frac{.1 \times f_{2}}{.534}=0$
$.035 f_{1}+.187 f_{2}=0$
$\frac{1}{f_{1}}+\frac{1}{f_{2}}=\frac{1}{40}$
$.035 f_{1}+.18 f_{2}=0$
$\frac{1}{f_{1}}+\frac{1}{-\frac{.035}{.187} f_{1}}=\frac{1}{40}$
$\frac{1}{f_{1}}-\frac{.187}{.035 f_{1}}=\frac{1}{40}$
$\frac{1}{f_{1}}\left[1-\frac{.187}{.035}\right]=\frac{1}{40}$
$\frac{1}{f_{1}}\left[\frac{.035-.187}{.035}\right]=\frac{1}{40}$
$\frac{1}{f_{1}}-\frac{5.34}{f_{1}}=\frac{1}{40}$
$\frac{1}{f_{1}}-[1-5.34]=\frac{1}{40}$
$\frac{1}{f_{1}}=\frac{1}{-40 \times 4.34}$
$f_{1}=173.6 \mathrm{~cm}$
$f_{2}=\left(\frac{.035}{.187} \times f_{1}\right)$
$=+32.49 \mathrm{~cm}$
4. Obtain an expression for the magnetic moment associated with a solenoid of length $L$ and number of turns $N$ carrying current $I$. The inner and outer radii of the solenoid are $r_{1}$ and $r_{2}$.

Sol.


$$
\begin{aligned}
& M=\int_{r_{2}}^{r_{1}} \mu_{0} N^{2} \frac{d x I}{\pi\left(r_{2}^{2}, 2\right)} \\
& =\frac{\mu_{0} N^{2} I}{\pi_{2}^{2}-r_{1}^{2}} \frac{\left(\pi_{2}^{3}-r_{1}^{3}\right)}{3}=\frac{\mu_{0} I N^{2}\left(r_{2}^{3}-r_{1}^{3}\right)}{3\left(r_{2}^{2}-r_{1}^{2}\right)}
\end{aligned}
$$

5. One end of a string is attached to a rigid wall point $O$, passes over a smooth pulley and carries a hanger $S$ of mass $M$ at its other end. Another object $P$ of mass $M$ is suspended from a light ring that can slide without friction, along the string as shown in the figure. OA is horizontal. Find the additional mass to be attached to the hanger $S$ so as to raise the object $P$ by 10 cm .


Ans. $m=\frac{2 \sqrt{13}-7}{8-2 \sqrt{13}} M$

Sol.

$\cos \theta=\frac{1}{2}$
$\theta=60^{\circ}$
$\ell_{1}=40 \mathrm{~cm}$
When block moves 10 cm in upward direction block S moves a distance
of $=(80-20 \sqrt{13}) \mathrm{cm}$
Now By energy conservation
$\mathrm{Mg} \times 10=(\mathrm{M}+\mathrm{m}) \mathrm{g}[80-20 \sqrt{13}]$
$m=\frac{2 \sqrt{13}-7}{8-2 \sqrt{13}} M$

## Physical constants you may need -

1. Charge on electron $\mathrm{e}=1.6 \times 10^{-19} \mathrm{C}$
2. Mass of an electron $\mathrm{m}_{\mathrm{e}}=9.1 \times 10^{-31} \mathrm{~kg}$
3. Universal gravitational constant $\mathrm{G}=6.67 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}$
4. Permittivity of free space $\varepsilon_{0}=8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \mathrm{m}^{2}$
5. Gas constant $\mathrm{R}=8.31 \mathrm{~J} / \mathrm{K} \mathrm{mol}$
6. Planck constant $\mathrm{h}=6.62 \times 10^{-34} \mathrm{Js}$
7. Stefan constant $\sigma=5.67 \times 10^{-8} \mathrm{~W} / \mathrm{m}^{2} \mathrm{k}^{4}$
8. Boltzmann constant $\mathrm{k}=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$

## INSTRUCTION TO CANDIDATES

1. The answers / solutions to this question paper will be available on our website - www.iapt.org.in by 3 rd December 2011.

## EVALUTION PROCEDURE (NSEP) :

2. Part ' $A$ ' Answers of ALL the candidates are examined.
3. Part A is evaluated of only those students who get marks above a certain "cut off" marks in Part A. (e.g. NSEP Total marks for Part A are 180. Students getting (say) 100 or more than 100 marks in Part A are identified and their Part B is evaluated. Thus "cut off" marks are 100 in this example.)
4. PART B IS NOT EVALUATED OF ALL THE CANDIDATES.

## CERTIFICATES \& AWARDS

Following certificates are awarded by the I.A.P.T. to students successful in NSEP.
i) Certficates to "Centre top 10\%" students on the basis of marks in part A only.
ii) Merit certificates to statewise Top 1\% students on the basis of ( $\mathrm{A}+\mathrm{B}$ ) marks.
iii) Merit certificate and prize in the form of a book to Nationwise Top 1\% students based on ( $\mathrm{A}+\mathrm{B}$ ) marks.
5. Result sheets and the "centre top 10\%" certificates of NSEP are dispatched to the Professor in charge of the centre. Thus you will get your marks from the Professor in charge of your centre by January 2011 end.
6. TOP 300 (or so) students are called for the next examination - Indian Natiqnal Physics Olympiads (INPhO). Individual letters are sent to these students ONLY.
7. Gold medals may be awarded to TOP 35 students in this entire process
8. No querries will be entertained in this regared.

